Multiple Steklov eigenvalues in a domain with a small hole

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Let Ω^o be a bounded open domain of $\mathbb{R}^n$. Let ν_{Ω^o} denote the outward unit normal to $\partial\Omega^o$. We assume that the Steklov problem $\Delta u = 0$ in Ω^o, $\frac{\partial u}{\partial \nu_{\Omega^o}} = \lambda u$ on $\partial\Omega^o$ has a multiple eigenvalue $\tilde{\lambda}$ of multiplicity r. Then we consider an annular domain $\Omega(\epsilon)$ obtained by removing from Ω^o a small cavity of size $\epsilon > 0$, and we show that under appropriate assumptions each elementary symmetric function of r eigenvalues of the Steklov problem $\Delta u = 0$ in $\Omega(\epsilon)$, $\frac{\partial u}{\partial \nu_{\Omega(\epsilon)}} = \lambda u$ on $\partial\Omega(\epsilon)$ which converge to $\tilde{\lambda}$ as ϵ tends to zero, equals real a analytic function defined in an open neighborhood of $(0,0)$ in $\mathbb{R}^2$ and computed at the point $(\epsilon, \delta_{2,n}\epsilon\log\epsilon)$ for $\epsilon > 0$ small enough. Here $\nu_{\Omega(\epsilon)}$ denotes the outward unit normal to $\partial\Omega(\epsilon)$, and $\delta_{2,2} \equiv 1$ and $\delta_{2,n} \equiv 0$ if $n \geq 3$. Such a result is an extension to multiple eigenvalues of a previous result obtained for simple eigenvalues in collaboration with S. Gryshchuk.

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